## On the Single-Point Determination of Intrinsic Viscosity

It is convenient to measure the intrinsic viscosity of a polymer solution [ $\eta$ ] from a single-point determination using equations proposed in the literature. ${ }^{1,2}$ However, we have been worried about the accuracy of this procedure as compared with the multipoint method based on Huggins' equation:

$$
\begin{equation*}
\eta_{\mathrm{sp}}=[\eta] c+K^{\prime}[\eta]^{2} c^{2} \tag{1}
\end{equation*}
$$

where the specific viscosity, $\eta_{\mathrm{sp}}$, is the relative viscosity $\eta_{\mathrm{r}}-1$ at concentration $c$, and $K^{\prime}$ is Huggins' constant.

As pointed out by Palit and Kar, ${ }^{2}$ various single-point equations can be derived from eq. (1) using the expansion

$$
\begin{equation*}
\ln \eta_{\mathrm{r}}=\ln \left(1+\eta_{\mathrm{sp}}\right)=\eta_{\mathrm{sp}}-1 / 2 \eta_{\mathrm{sp}}^{2}+1 / 3 \eta_{\mathrm{sp}}^{3}-1 / 4 \eta_{\mathrm{sp}}{ }^{4} \tag{2}
\end{equation*}
$$

if $K^{\prime}$ is assumed to be zero.
By neglecting terms above quadratic in eq. (2), they deduced the equation proposed by Solomon and Ciuta ${ }^{1}$ :

$$
\begin{equation*}
[\eta]=\frac{1}{c}\left(2 \eta_{\mathrm{sp}}-2 \ln \eta_{\mathrm{r}}\right)^{1 / 2} . \tag{3}
\end{equation*}
$$

However if one substitutes $1+\eta_{\mathrm{sp}}$ for $\eta_{\mathrm{r}}$ in eq. (3), then expansion gives

$$
\begin{equation*}
[\eta] c=\eta_{\mathrm{sp}}\left(1-2 / 3 \eta_{\mathrm{sp}}\right)^{1 / 2} \tag{4}
\end{equation*}
$$

and expanding the square root and solving for $\eta_{\mathrm{sp}}$ gives

$$
\begin{equation*}
\eta_{\mathrm{sp}}=\frac{3-(9-12[\eta] c)^{1 / 2}}{2} \tag{5}
\end{equation*}
$$

which upon expansion of the square root leads to

$$
\begin{equation*}
\eta_{\mathrm{sp}}=[\eta] c+1 / 3[\eta]^{2} c^{2} \tag{6}
\end{equation*}
$$



Fig. 1. A comparison of the intrinsic viscosity determined by the single-point method of Solomon and Cuita and the multipoint method using the Huggins equation.

The single-point method using eq. (3) should therefore give results in agreement with Huggins' equation when $K^{\prime}=1 / 3$. For values of $K^{\prime}>1 / 3$, this single-point method gives results for $[\eta]$ denoted by $[\eta]_{s c}$ which are higher than the values obtained by the multipoint method $[\eta]_{\mathrm{n}}$. For $K^{\prime}<1 / 3,[\eta]_{\mathrm{sc}}$ is less than $[\eta]_{\mathrm{H}}$. The ratio $[\eta]_{\mathrm{so}} /[\eta]_{\mathrm{H}}$ is illustrated in Figure 1 as a function of $\eta_{\mathrm{sp}}$.

By assuming $K^{\prime}=0$ and including the cubic term in eq. (2), Palit and Kar ${ }^{2}$ deduced the single-point equation proposed by Deb and Chatterjee. ${ }^{3}$ By algebraic expansion as outlined above it can be shown that the Deb and Chatterjee equation should agree with the Huggins equation when $K^{\prime}=1 / 4$. Similarly by including the biquadratic terms in eq. (2), a single-point equation is obtained which agrees with the Huggins equation when $K^{\prime}=1 / 5$. Thus the divergence between the single-point and multipoint methods will become larger as more terms are included in eq. (2) provided $K^{\prime}>1 / 3$; it is not necessarily due to errors in measuring $\eta_{\mathrm{sp}}$ as suggested by Palit and Kar. ${ }^{2}$ Clearly, the value of $K^{\prime}$ must be known in order to select the single-point equation in best agreement with Huggins' equation.

## References

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2. S. R. Palit and I. Kar, J. Polym. Sci. A-1, 5, 2629 (1967).
3. P. C. Deb and S. R. Chatterjee, unpublished work cited in ref. 2.
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